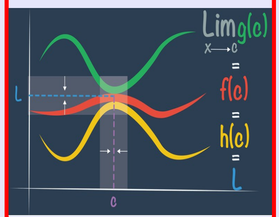
Calculus I

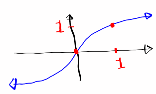
Lecture 35



Feb 19-8:47 AM

$f(x)=\sqrt[3]{x}$, $[0,1]$

odd index $\to$ Domain $(-\infty,\infty)$



$f(-x)=\sqrt[3]{-x}=-\sqrt[3]{x}=-f(x)$

odd function

symmetric w/t origin

1) $f(x)$ is cont. on $[0,1]$

$f(x)=x^{1/3}$ $\quad f'(x)=\frac{1}{3}x^{-2/3}=\frac{1}{3\sqrt[3]{x^2}}$

2) $f(x)$ is diff. on $(0,1)$

By MVT, there is at least a number c in $(0,1)$ such that

$$f'(c)=\frac{f(1)-f(0)}{1-0}$$

$$\frac{1}{3\sqrt[3]{c^2}}=\frac{1-0}{1-0}$$

$$\frac{1}{3\sqrt[3]{c^2}}=1 \to 3\sqrt[3]{c^2}=1$$

$$\sqrt[3]{c^2}=\frac{1}{3}$$

⋮

Solve for c

$c^2=\left(\frac{1}{3}\right)^3$

$c^2=\frac{1}{27}$ $\quad c=\pm\sqrt{\frac{1}{27}}$

$c=\pm\frac{\sqrt{27}}{27}$ $\quad c=\pm\frac{3\sqrt{3}}{27}$ $\quad c=\pm\frac{\sqrt{3}}{9}$

focus on $(0,1)$

$c=\frac{\sqrt{3}}{9}$ $\quad c\approx .192 \to$ it is in $(0,1)$

Rolle's Thrm , MVT

Apr 16-8:52 AM

$f(x) = \dfrac{x^2}{x^2+3}$

$x^2+3 \neq 0 \rightarrow$ Domain $(-\infty, \infty)$

No Vertical Asymptote

$\lim\limits_{x\to\infty} \dfrac{x^2}{x^2+3} = 1$

$\lim\limits_{x\to-\infty} \dfrac{x^2}{x^2+3} = 1$ $\Rightarrow$ H.A. $y=1$

$f(-x) = \dfrac{(-x)^2}{(-x)^2+3} = \dfrac{x^2}{x^2+3} = f(x) \Rightarrow$ f(x) is an even function, Symmetric w/t Y-axis.

$f(0) = \dfrac{0^2}{0^2+3} = \dfrac{0}{3} = 0 \rightarrow$ x-Int, Y-Int $\rightarrow (0,0)$

Find $f'(x)$ & $f''(x)$

$f(x) = \dfrac{x^2+3-3}{x^2+3}$ $\quad f(x) = \dfrac{x^2+3}{x^2+3} - \dfrac{3}{x^2+3} = 1 - 3(x^2+3)^{-1}$

$f'(x) = 3(x^2+3)^{-2} \cdot 2x$ $\quad \boxed{f'(x) = \dfrac{6x}{(x^2+3)^2}}$

$f''(x) = 6\left[1 \cdot (x^2+3)^{-2} + x \cdot -2(x^2+3)^{-3} \cdot 2x\right] = 6(x^2+3)^{-3}\left[(x^2+3)^1 - 4x^2\right]$

$= \dfrac{6(3-3x^2)}{(x^2+3)^3} = \dfrac{-18(x^2-1)}{(x^2+3)^3} = \dfrac{-18(x+1)(x-1)}{(x^2+3)^3}$

Apr 16-9:01 AM

$f'(x) = \dfrac{6x}{(x^2+3)^2}$ $\qquad f''(x) = \dfrac{-18(x-1)(x+1)}{(x^2+3)^3}$

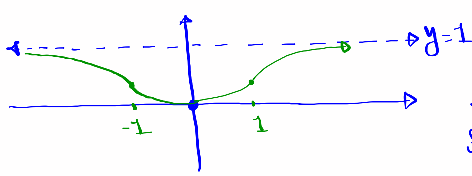
$f'(x) = 0 \rightarrow 6x = 0 \rightarrow x = 0$

$f'(x)$ is never undefined.

$f''(x) = 0 \rightarrow -18(x-1)(x+1) = 0 \rightarrow x = \pm 1$

$f''(x)$ is never undefined.

x	$-\infty$ … -1	-1 … 0	0 … 1	1 … ∞
$f'(x)$	$-$	$-$	$+$	$+$
$f''(x)$	$-$	$+$	$+$	$-$
$f(x)$	Dec., C.D.	Dec., C.U.	Inc. C.U	Inc., C.D
		I.P.		I.P.



$f(1) = \frac{1}{4}$

$f(-1) = \frac{1}{4}$

Apr 16-9:16 AM

Besides graphing, How do we use these concepts?

x & y

Find $\boxed{\text{two numbers}}$ such that their sum is 20,

and $\boxed{\text{their product}}$ is maximum.

$x \cdot y$

$x + y = 20$

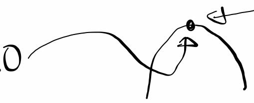
$y = 20 - x$

Product $x(20-x)$

$f(x) = 20x - x^2$

Parabola, opens downward

$f'(x) = 20 - 2x$ $\quad f'(x) = 0 \rightarrow x = 10$



$f''(x) = -2 < 0 \rightarrow$ Concave down

Max. Prod. is at $x = 10$

Two numbers are 10 & $20 - 10$

10 & 10

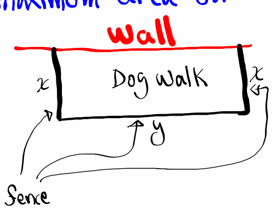
Apr 16-9:26 AM

A farmer has $\boxed{\text{200 ft of fencing}}$

He wants to build a rectangular dog walk

next to an existing wall.

what dimensions gives maximum area for the dog walk?



Total fence $\rightarrow 2x + y = 200$

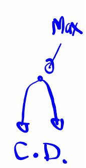
Area $\rightarrow xy \rightarrow x(200 - 2x)$

Let $f(x) = x(200 - 2x)$

$f(x) = 200x - 2x^2$

$f'(x) = 200 - 4x$

$f''(x) = -4 < 0$



$f'(x) = 0$

$200 - 4x = 0$

$x = 50$

$y = 100$

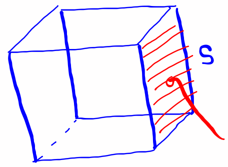
Dimensions are

50 ft by 100 ft

Max. Area $\quad 5000\ ft^2$

Apr 16-9:32 AM

The volume of a cube is increasing at a rate of $10\ cm^3/min$.



$V = S^3$

$\frac{dV}{dt} = 10\ cm^3/min$

How fast is the surface area increasing when the length of the side is 30 cm?

$A = 6S^2$

$\frac{dA}{dt} = ?$ when $S = 30$

$V = S^3$ $\quad \frac{dV}{dt} = 3S^2 \frac{dS}{dt}$

$10 = 3 \cdot 30^2 \frac{dS}{dt}$

$\frac{dA}{dt} = 12S \frac{dS}{dt}$

$\frac{dS}{dt} = \frac{10}{2700}$

$\frac{dS}{dt} = \frac{1}{270}$ cm/min

$\frac{dA}{dt} = \cancel{12}^{4} \cdot \cancel{30} \cdot \frac{1}{\cancel{270}_{\cancel{9}_{3}}}$

$= \boxed{\frac{4}{3}\ cm^2/min}$

Apr 16-9:40 AM

Evaluate $\boxed{\lim_{x \to \pi/3} \frac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}}} = \frac{\cos\frac{\pi}{3} - \frac{1}{2}}{\pi/3 - \pi/3}$

$= \frac{\cos 60^\circ - \frac{1}{2}}{\frac{\pi}{3} - \frac{\pi}{3}} = \frac{\frac{1}{2} - \frac{1}{2}}{\frac{\pi}{3} - \frac{\pi}{3}}$

$= \frac{0}{0}$ I.F.

$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$

$f(x) = \cos x$

$a \to \frac{\pi}{3}$

$\to = \frac{d}{dx}\left[\cos x\right]\Big|_{x = \frac{\pi}{3}}$

$= -\sin\frac{\pi}{3} = -\sin 60^\circ = \boxed{-\frac{\sqrt{3}}{2}}$

Exam 3 → May 2, 2024

Apr 16-9:50 AM